
THE IMPACT OF FAKE NEWS IN MODERN POLITICAL CAMPAIGNS

A MATHEMATICAL APPROACH



Peter Dazeley Getty Images

H Aidong Ji

Zeus Garyulo

July 16, 2018

Summary

The purpose of this report is to study the impact of fake news in modern political campaigns, a subject that has attracted a lot of attention since the last U.S. presidential election in 2016.

Starting with a classic compartmental model (the SIR model), using Python as the programming language and Euler's method as the procedure for solving the system of ordinary differential equations, a set of five countries have been selected to analyse how fake news distorts public opinion.

From this research, we can establish that fake news definitely impact the outcome of a political campaign, but maybe not as much as we think, education and freedom of speech are powerful tools to fight this phenomenon.

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List of variables

| Symbol | Description | Units |
|------------|-------------------------------------|--------------------|
| $P(t)$ | Population involved in the election | Number of citizens |
| $S(t)$ | Susceptible population fraction | Dimensionless |
| $I(t)$ | Infected population fraction | Dimensionless |
| $R(t)$ | Recovered population fraction | Dimensionless |
| α | Rate of recovery | $1/t$ |
| β | Contact rate | $1/t$ |
| R_0 | Basic reproduction number | Dimensionless |
| T_c | Typical time between contacts | t (days) |
| T_r | Typical time until recovery | t (days) |
| $dP(t)/dt$ | Rate of change of $P(t)$ | $1/t$ |
| $dS(t)/dt$ | Rate of change of $S(t)$ | $1/t$ |
| $dI(t)/dt$ | Rate of change of $I(t)$ | $1/t$ |
| $dR(t)/dt$ | Rate of change of $R(t)$ | $1/t$ |

Chapter 1

Introduction

Fake news, or fabricated content deceptively presented as real news, has garnered a lot of interest since the 2016 U.S. presidential election.

Although hardly a new phenomenon, the global nature of the web-based information environment allows purveyors of all sorts of falsehoods and misinformation to make an international impact. As a result, we talk of fake news and its impact not only in the United States, but also in France, Italy and the United Kingdom.

Even though the rise of fake news in recent months is undeniable, its impact is a different story. The persuasive effects of these stories have not been quantified (yet).

The aim of this paper is to shed light on the impact of fake news in modern political campaigns. In recent years, “fake news” – articles that are intentionally and verifiably false and could mislead readers – has contributed to an increasingly uncertain political climate.

The problem has been modelled modifying the classical compartmental models used to predict the propagation of an infectious disease[1]. The simulation shows that societies with a high rate of internet penetration[2] are more vulnerable to propagation of fake news. Additionally, countries with low HDI (human developed index) show a longer lasting impact of fake news, polarizing societies even more. This paper provides a backdrop against which effective policy response can be designed to tackle the growing problem of fake news.

Regarding the structure of this report, chapter 2 provides some historical context to the rising issue of fake news around the globe in the last five years, chapter 3 introduces the mathematical model developed in detail, while chapter 4 is a summary of the results obtained for each country.

For the more curious readers, appendix A contains a complete explanation of Euler’s method, appendix B the Python’s code developed and appendixes C, D the raw-data used to run the simulations.

Chapter 2

Fake news around the world

In this chapter we will provide some (recent) historical context to the global problem of fake news, many countries around the world have experienced unexpected (and controversial) results in the last five years elections[4].

2.1 United States of America

Fake news became a global subject and was widely introduced to billions as a subject mainly due to the 2016 U.S. presidential election. Numerous political commentators and journalists wrote and stated in media that 2016 was the year of fake news and as a result nothing will ever be the same in politics and cyber security. Due to the fair amount of fake news in 2016, it became hard to tell what was real in 2017. Donald Trump tweeted or retweeted posts about "fake news" or "fake media" 176 times as of Dec. 20, 2017, according to an online archive of all of Trump's tweets. Governmental bodies in the U.S. and Europe started looking at contingencies and regulations to combat fake news specially when as part of a coordinated intelligence campaign by hostile foreign governments. Online tech giants Facebook and Google started putting in place means to combat fake news in 2016 as a result of the phenomenon becoming globally known. Google Trends shows that the term "fake news" gained traction in online searches in October 2016.

2.2 United Kingdom

Brexit is the impending withdrawal of the United Kingdom (UK) from the European Union (EU). In a referendum on 23 June 2016, 51.9% of the participating UK electorate voted to leave the EU, out of a turnout of 72.2%. On 29 March 2017, the UK government invoked Article 50 of the Treaty on the European Union. The UK is thus due to leave the EU at 11 pm on 29 March 2019 UTC.

Along the campaign, several moments can easily be identified as fake news, to name a few:

- Farage's infamous "Breaking Point" poster can be described as a "fake" since it showed a queue of migrants at the Croatia-Slovenia border, not trying to get into Britain. It was

denounced by Vote Leave.

- Vote Leave was certainly on the border between false and fake news. One of its posters claimed: “Turkey (population 76 million) is joining the EU.” Penny Mordaunt, a Defence Minister, claimed the Government would not be able to stop Turkish criminals entering the UK or to veto Turkey’s EU accession (the latter a downright lie).
- The ultimate piece of fake news was the claim that leaving would provide a £350m-a-week bonus for the NHS from the UK’s contribution to EU coffers.

2.3 Ukraine

Since the Euromaidan and the beginning of the Ukrainian crisis in 2014, the Ukrainian media circulated several fake news stories and misleading images, including a dead rebel photograph with a Photoshop-painted tattoo which allegedly indicated that he belonged to Russian Special Forces, a video game screenshot disguised as a satellite image ostensibly showing the shelling of the Ukrainian border from Russia, and the threat of a Russian nuclear attack against the Ukrainian troops. The recurring theme of these fake news was that Russia was solely to blame for the crisis and the war in Donbass.

2.4 Spain

The topic of fake news has traditionally not been given much attention in Spain, until the newspaper *El País* launched the new blog dedicated strictly to truthful news entitled “Hechos”; which literally translates to “fact” in Spanish. David Alandete, the managing editor of *El País*, stated how many people misinterpret fake news as real because the sites “have similar names, typography, layouts and are deliberately confusing”. Alandete made it the new mission of *El País* “to respond to fake news”. Most recently *El País* has created a fact-checking position for five employees, to try and debunk the fake news released.

2.5 France

During the 10-year period preceding 2016, France was witness to an increase in popularity of far-right alternative news sources called the fachosphere (“facho” referring to fascist); known as the extreme right on the Internet.

In September 2016, the country faced controversy regarding fake websites providing false information about abortion. The National Assembly moved forward with intentions to ban such fake sites. Laurence Rossignol, women’s minister for France, informed parliament though the fake sites look neutral, in actuality their intentions were specifically targeted to give women fake information.

France saw an uptick in amounts of disinformation and propaganda, primarily in the midst of election cycles. Social media outlets in France were overflowing with fake news prior to the 2017 presidential election. A study looking at the diffusion of political news during the 2017

presidential election cycle suggests that one in four links shared in social media comes from sources that actively contest traditional media narratives.

2.6 Brazil

Brazil faced increasing influence from fake news after the 2014 re-election of President Dilma Rousseff and Rousseff's subsequent impeachment in August 2016. BBC Brazil reported in April 2016 that in the week surrounding one of the impeachment votes, three out of the five most-shared articles on Facebook in Brazil were fake. In 2015, reporter Tai Nalon resigned from her position at Brazilian newspaper *Folha de S.Paulo* in order to start the first fact-checking website in Brazil, called *Aos Fatos* (To the Facts).

Chapter 3

Mathematical model

Compartmental models are a technique used to simplify the mathematical modelling of infectious disease. The population is divided into compartments, with the assumption that every individual in the same compartment has the same characteristics. The origin of this method date from the beginning of the 20th century[1].

The dynamics of an epidemic, for example the flu, are often much faster than the dynamics of birth and death, therefore, birth and death are often omitted in simple compartmental models. The SIR system¹ without so-called vital dynamics (birth and death, sometimes called demography) described above can be expressed by the following set of ordinary differential equations:

$$\begin{cases} \frac{dS(t)}{dt} = -\beta S(t)I(t), \\ \frac{dI(t)}{dt} = \beta S(t)I(t) - \alpha I(t), \\ \frac{dR(t)}{dt} = \alpha I(t). \end{cases} \quad (3.1)$$

Where:

| Symbol | Description | Units |
|------------|---------------------------------|---------------|
| $S(t)$ | Susceptible population fraction | Dimensionless |
| $I(t)$ | Infected population fraction | Dimensionless |
| $R(t)$ | Recovered population fraction | Dimensionless |
| α | Rate of recovery | $1/t$ |
| β | Contact rate | $1/t$ |
| $dS(t)/dt$ | Rate of change of $S(t)$ | $1/t$ |
| $dI(t)/dt$ | Rate of change of $I(t)$ | $1/t$ |
| $dR(t)/dt$ | Rate of change of $R(t)$ | $1/t$ |

¹Notice that this system is non-linear, and does not admit a generic analytic solution.

3.1 SIR model

The SIR model is one of the simplest compartmental models, and many models are derivations of this basic form. The model consists of three compartments:

- S for the number of susceptible people,
- I for the number of infectious people,
- R for the number of recovered (or immune) people.

This model is reasonably predictive for infectious diseases which are transmitted from human to human, and where recovery confers lasting resistance, such as measles, mumps and rubella.

The variables (S, I, and R) represent the number of people in each compartment at a particular time. To represent that the number of susceptible, infected and recovered individuals may vary over time (even if the total population size remains constant), we make the precise numbers a function of t (time): S(t), I(t) and R(t). For a specific disease in a specific population, these functions may be worked out in order to predict possible outbreaks and bring them under control.

3.1.1 Model dynamics

The dynamics of the infectious class depends on the following ratio:

$$R_0 = \frac{\beta}{\alpha}$$

Which is known as the basic reproduction number (also called basic reproduction ratio). This ratio is derived as the expected number of new infections (these new infections are sometimes called secondary infections) from a single infection in a population where all subjects are susceptible. This idea can probably be more readily seen if we say that the typical time between contacts is $T_c = \beta^{-1}$, and the typical time until recovery is $T_r = \gamma^{-1}$. From here it follows that, on average, the number of contacts by an infected individual with others before the infected has recovered is: T_r/T_c .

Taking into account the purpose of this study (to compare how fake news impact in different countries), we have linked β and α to important indexes commonly used to describe the social, economical and cultural performance of societies.

Specifically:

$$\begin{aligned}\beta &= INT/10 \\ \alpha &= HDI/100\end{aligned}$$

Where INT is an index describing the internet penetration of a given country[2], and HDI is the Human Development Index[3].

Because it is easier to spread a lie than reaffirming a truth, the value of α is smaller than the value of β .

3.1.2 Population is constant in the mathematical model

We can define the total population participating in the election as:

$$P(t) = S(t) + I(t) + R(t) + V(t) \quad (3.2)$$

Where we have introduced $V(t)$ as the fraction of voters that will never change their minds regarding which candidate they will support, in political jargon, these are called "hard core voters".²

Now, in our model we will consider that the number of voters stays constant during the political cycle, this means that $\frac{dP(t)}{dt}$ should be zero, is this true?

Proof. Let's divide both sides of 3.2 by Δt :

$$\frac{P(t)}{\Delta t} = \frac{S(t)}{\Delta t} + \frac{I(t)}{\Delta t} + \frac{R(t)}{\Delta t} + \frac{V(t)}{\Delta t}$$

Taking limits

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \frac{P(t)}{\Delta t} &= \lim_{\Delta t \rightarrow 0} \left(\frac{S(t)}{\Delta t} + \frac{I(t)}{\Delta t} + \frac{R(t)}{\Delta t} + \frac{V(t)}{\Delta t} \right) \\ \frac{dP(t)}{dt} &= \frac{dS(t)}{dt} + \frac{dI(t)}{dt} + \frac{dR(t)}{dt} + \frac{dV(t)}{dt} \end{aligned}$$

Substituting

$$\frac{dP(t)}{dt} = -\beta S(t)I(t) + \beta S(t)I(t) - \alpha I(t) + \alpha I(t) + \frac{dV(t)}{dt}$$

Reagrouping terms

$$\begin{aligned} \frac{dP(t)}{dt} &= \overbrace{(-\beta S(t)I(t) + \beta S(t)I(t))}^{=0} + \overbrace{(\alpha I(t) - \alpha I(t))}^{=0} + \overbrace{\frac{dV(t)}{dt}}^{=0} \\ \frac{dP(t)}{dt} &= 0 \Rightarrow P(t) \text{ is a constant function.} \end{aligned}$$

□

²Notice that $V(t)$ doesn't intervene in 3.1.

Chapter 4

Results

We will apply the mathematical model to the following countries:

- Argentina
- Italy
- Mozambique
- Norway
- Vietnam

Table 4.1 shows the different parameters and constants used in Python to run the simulations. The code has been included in appendix B, it is based on Euler's method, a basic explanation of the algorithm is given in appendix A.

Table 4.1: **Input parameters for each country**

| Country | α | β | R_0 | HDI | INT |
|------------|----------|---------|-------|-------|-------|
| Argentina | 0.008 | 0.07 | 8.75 | 0.827 | 0.702 |
| Italy | 0.009 | 0.061 | 6.78 | 0.887 | 0.613 |
| Mozambique | 0.004 | 0.018 | 4.50 | 0.418 | 0.175 |
| Norway | 0.009 | 0.097 | 10.78 | 0.949 | 0.973 |
| Vietnam | 0.007 | 0.046 | 6.57 | 0.683 | 0.465 |

4.1 Argentina

| Input parameters | | | | |
|------------------|---------|-------|-------|-------|
| α | β | R_0 | HDI | INT |
| 0.008 | 0.07 | 8.75 | 0.827 | 0.702 |

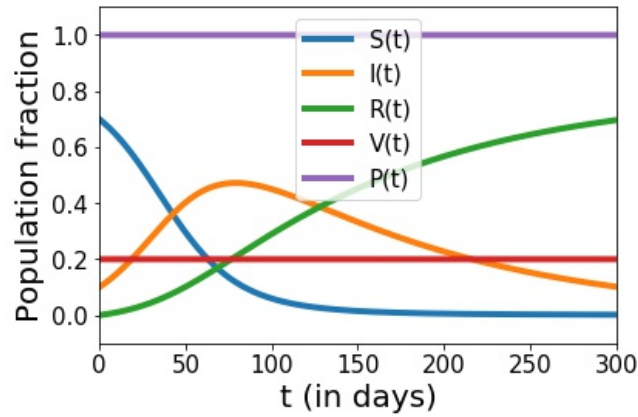


Figure 4.1: Results for **Argentina**

Short term impact: The relative high INT of Argentina allows fake news to travel fast around the population, around 80 days we have a maximum of infected people, which indicates that timed properly, this could have an important influence in the result of a political campaign.

Long term impact: The high HDI of the country neutralizes the effect of the campaign, but it takes around 300 days to fully return to the original distribution of infected population.

4.2 Italy

| Input parameters | | | | |
|------------------|---------|-------|-------|-------|
| α | β | R_0 | HDI | INT |
| 0.009 | 0.061 | 6.78 | 0.887 | 0.613 |

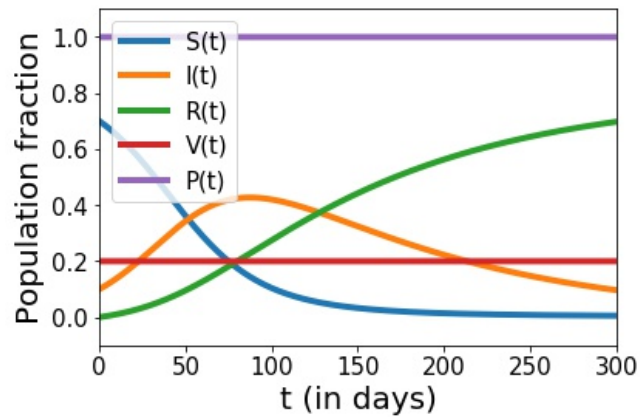


Figure 4.2: Results for **Italy**

Short term impact: For Italy we have a very similar behaviour to that of Argentina, this is explained by the similar input parameters for both countries.

Long term impact: Again, it is surprising to see how long it takes for a society to recover its original compartment distribution before a fake news operation, even for a modern European democracy.

4.3 Mozambique

Input parameters

| α | β | R_0 | HDI | INT |
|----------|---------|-------|-------|--------|
| 0.004 | 0.018 | 4.50 | 0.418 | 0.1752 |

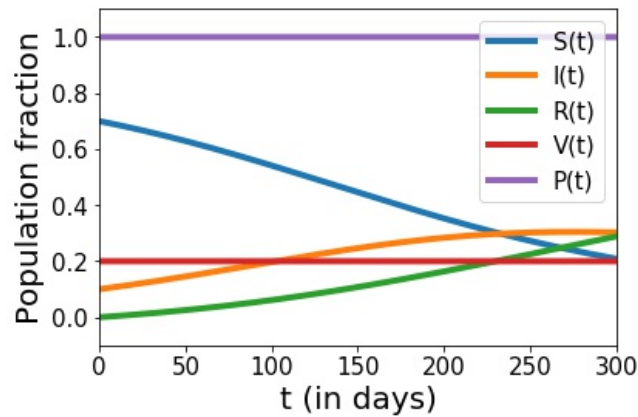


Figure 4.3: Results for **Mozambique**

Short term impact: The low INT is reflected in the graph as a very smooth and slow increase in the number of infected citizens, having a very small impact in the period before an election (less than three months).

Long term impact: On the other hand, the also low HDI index shows that is very difficult to recover the original population distribution along the compartments, for the first time we see a long lasting impact of fake news in the political spectrum.

4.4 Norway

| Input parameters | | | | |
|------------------|---------|-------|-------|-------|
| α | β | R_0 | HDI | INT |
| 0.009 | 0.097 | 10.78 | 0.949 | 0.973 |

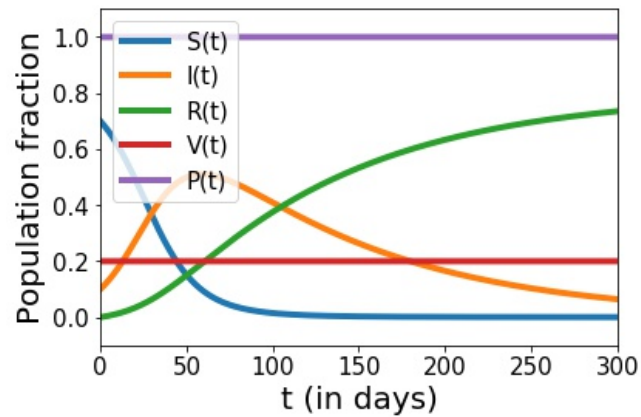


Figure 4.4: Results for **Norway**

Short term impact: It is not a surprise that Norway has a very high INT factor (the right to internet access is protected by the State), the results show the fastest spread rate for fake news for all countries analysed.

Long term impact: At the same time, the also high HDI index allows a quick recovery of the population, with no long term impact appreciable.

4.5 Vietnam

| Input parameters | | | | |
|------------------|---------|-------|-------|-------|
| α | β | R_0 | HDI | INT |
| 0.007 | 0.046 | 6.57 | 0.683 | 0.465 |

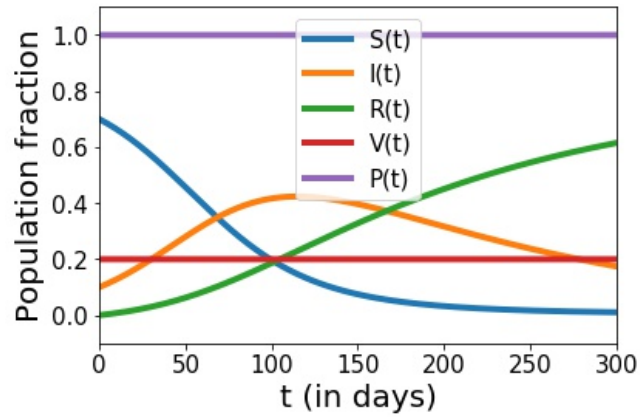


Figure 4.5: Results for **Vietnam**

Short term impact: The low INT of the country protects the country against the effect of a fake news campaign, with negligible impact in the months prior to an election.

Long term impact: Even after a year, the original distribution hasn't been recovered, the combination of low INT and low HDI produces the particular bell shaped curve for the infected population.

4.6 Discussion

The results are interesting and surprising, even for well developed countries like Norway and Italy, the impact of fake news is very noticeable, the high rate of internet penetration along these countries makes them particularly vulnerable to this new phenomenon.

On the other hand, we find that for under-developed countries the impact in the short term is not so important, but taking into account the prediction for the evolution of internet users in the next twenty years, they could soon be victims of this new way of political propaganda.

We should not forget that our mathematical model is very limited in its nature, behavioural dynamics is a very complex topic, we have only taken into account two parameters (HDI and INT), but many others factors play a role determining how easily will someone believe what it reads or hears.

Nevertheless, the model presented has to be taken as a starting point in our effort to quantify the evolution and impact of fake news in modern democracies.

Chapter 5

Conclusions

Fake news is a real problem, internet penetration around the globe will continue increasing steadily, the role of the web in modern democracies has to be discussed, mechanisms to stop the spread of bad information have to be developed in order to protect a central part of the democratic system, the election process.

Results show that even for societies with high HDI index, a well thought fake news campaign can have a severe impact in the outcome of an election, the mechanism to spread lies is much faster and efficient than the process of researching, checking and refuting fake news.

The flow of bad information helps to create a cultural chasm in the society, which paralyses the political system, destroys consensus and radicalises political parties.

The political leadership must elaborate better public policies to protect society and penalise those who want to gain an advantage through impure methods. The civil society needs to take conscience of the political impact of fake news and censor those information channels that damage the entire democratic ecosystem.

Both actors must act together if democracy wants to prevail as the main system of government in the XXI century, after all, as Churchill said:

"Democracy is the worst form of government, except for all the others."

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Appendix A

Euler's method

In mathematics and computational science, the Euler method is a first-order numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations and is the simplest Runge–Kutta method. The Euler method is named after Leonhard Euler, who treated it in his book *Institutionum calculi integralis* (published 1768–1870).

The Euler method is a first-order method, which means that the local error (error per step) is proportional to the square of the step size, and the global error (error at a given time) is proportional to the step size.

Suppose we wish to approximate the solution to the following initial-value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0,$$

At $x = x_1 = x_0 + h$, where h is small. The idea behind Euler's method is to use the tangent line to the solution curve through (x_0, y_0) to obtain such an approximation. The equation of the tangent line through (x_0, y_0) is

$$y(x) = y_0 + m(x - x_0),$$

where m is the slope of the curve at (x_0, y_0) , we know that $m = f(x_0, y_0)$, so

$$y(x) = y_0 + f(x_0, y_0)(x - x_0).$$

Setting $x = x_1$ in this equation yields the Euler approximation to the exact solution at x_1 , namely

$$y_1 = y_0 + f(x_0, y_0)(x_1 - x_0).$$

Which we write as

$$y_1 = y_0 + f(x_1, y_1)(x - x_1)$$

Setting $x = x_2$ yields the approximation

$$y_2 = y_1 + hf(x_1, y_1),$$

where we have substituted for $x_2 - x_1 = h$ to the solution to the initial-value problem at $x = x_2$. Continuing in this manner, we determine the sequence of approximations

$$y_{n+1} = y_n + hf(x_n, y_n), \quad n = 0, 1, \dots$$

to the solution to the initial-value problem at the points $x_{n+1} = x_n + h$.
 In summary, **Euler's method** for approximating the solution to the initial-value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0,$$

at the points $x_{n+1} = x_0 + nh$ ($n = 0, 1, \dots$) is

$$y_{n+1} = y_n + hf(x_n, y_n), \quad n = 0, 1, \dots$$

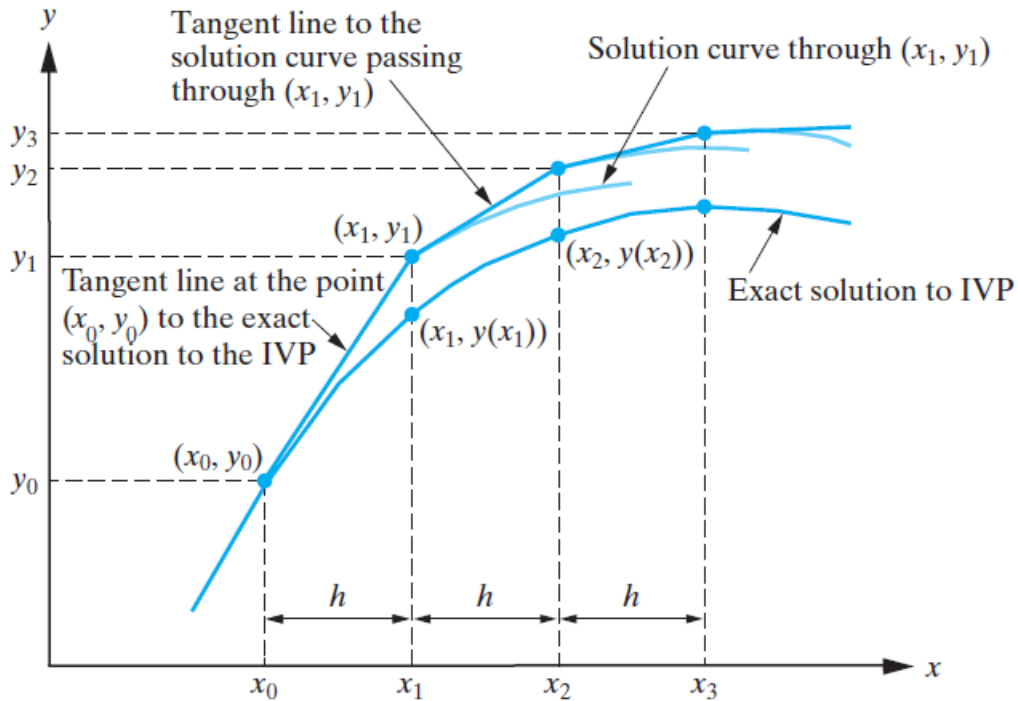


Figure A.1: *Euler's method for approximating the solution to the initial-value problem $dy/dx = f(x, y)$, $y(x_0) = y_0$,*

Appendix B

Python code

```
import numpy as np
import matplotlib.pyplot as plt

#Human Development Index
HDI = {"Mozambique":.418, "Norway":.949, "Italy":.887,
       "Argentina":.827, "Vietnam":.683}

for country in HDI:

    # Initializations
    Dt = .001 # timestep Delta t
    S_init = 0.7 # initial susceptible population
    I_init = 0.1 # initial infected population
    R_init = 0 # initial recovered population
    V_init = .20 # hard core voters
    P_init = S_init + I_init + R_init + V_init
    t_init = 0 # initial time
    t_end = 30 # stopping time

    n_steps = int(round((t_end-t_init)/Dt)) # total number of timesteps

    X = np.zeros(5) # create space for current X=[S,I,R,V,P]^T
    dXdt = np.zeros(5) # create space for current derivative
    t_arr = np.zeros(n_steps + 1) # create a storage array for t
    X_arr = np.zeros((5,n_steps+1)) # create a storage array for X=[P,G]^T
    t_arr[0] = t_init # add the initial t to the storage array
    X_arr[0,0] = S_init # add the initial S to the storage array
    X_arr[1,0] = I_init # add the initial I to the storage array
    X_arr[2,0] = R_init # add the initial R to the storage array
    X_arr[3,0] = V_init
    X_arr[4,0] = P_init

    betha = 1 - HDI[country]
    alpha = betha * .1
```

```

# Euler's method

for i in range (1, n_steps + 1):
    t = t_arr[i-1]           # load the time
    S = X_arr[0,i-1]        # load the value of S
    I = X_arr[1,i-1]        # load the value of I
    R = X_arr[2,i-1]        # load the value of R
    V = X_arr[3,i-1]        # load the value of R
    P = X_arr[4,i-1]        # load the value of R

    X[0] = S                # fill current state vector X=[S,I,R,V,P]^T
    X[1] = I                # fill current state vector X=[S,I,R,V,P]^T
    X[2] = R                # fill current state vector X=[S,I,R,V,P]^T
    X[3] = V                # fill current state vector X=[S,I,R,V,P]^T
    X[4] = S+I+R+V          # fill current state vector X=[S,I,R,V,P]^T

    dSdt = -beta*S*I        # calculate the derivative dS/dt
    dIdt = beta*S*I-alpha*I # calculate the derivative dR/dt
    dRdt = alpha*I          # calculate the derivative dR/dt
    dVdt = 0                # calculate the derivative dV/dt
    dPdt = dSdt + dIdt + dRdt + dVdt # calculate the derivative dV/dt

    dXdt[0] = dSdt          # fill derivative vector dS/dt
    dXdt[1] = dIdt          # fill derivative vector dI/dt
    dXdt[2] = dRdt          # fill derivative vector dR/dt
    dXdt[3] = dVdt          # fill derivative vector dR/dt
    dXdt[4] = dPdt          # fill derivative vector dR/dt

    Xnew = X + Dt*dXdt      # calculate X on next time step
    X_arr[:,i] = Xnew       # store Xnew
    t_arr[i] = t + Dt       # store new t-value

fig = plt.figure()
plt.plot(t_arr, X_arr[0,:], linewidth = 4, label="S(t)") # plot S vs. time
plt.plot(t_arr, X_arr[1,:], linewidth = 4, label="I(t)") # plot I vs. time
plt.plot(t_arr, X_arr[2,:], linewidth = 4, label="R(t)") # plot R vs. time
plt.plot(t_arr, X_arr[3,:], linewidth = 4, label="V(t)") # plot R vs. time
plt.plot(t_arr, X_arr[4,:], linewidth = 4, label="P(t)") # plot R vs. time

plt.title('Results', fontsize = 20) # set title
plt.xlabel('t (in days)', fontsize = 20) # name of horizontal axis
plt.ylabel('Population fraction', fontsize = 20) # name of vertical axis

plt.xticks(fontsize = 15) # adjust the fontsize
plt.yticks(fontsize = 15) # adjust the fontsize
plt.axis([0, t_end, -.1, 1.1 ]) # set the range of the axes

plt.legend(fontsize=15) # show the legend
plt.show() # necessary for some platforms

```

Appendix C

List of countries by Human Development Index

Extracted from [Wikipedia](#).

- | | | |
|--------------------------|--------------------------|--------------------------------|
| 1. Norway 0.949 | 20. Luxembourg 0.898 | 39. Saudi Arabia 0.847 |
| 2. Australia 0.939 | 21. France 0.897 | 40. Slovakia 0.845 |
| 3. Switzerland 0.939 | 22. Belgium 0.896 | 41. Portugal 0.843 |
| 4. Germany 0.926 | 23. Finland 0.895 | 42. United Arab Emirates 0.840 |
| 5. Denmark 0.925 | 24. Austria 0.893 | 43. Hungary 0.836 |
| 6. Singapore 0.925 | 25. Slovenia 0.890 | 44. Latvia 0.830 |
| 7. Netherlands 0.924 | 26. Italy 0.887 | 45. Argentina 0.827 |
| 8. Ireland 0.923 | 27. Spain 0.884 | 46. Croatia 0.827 |
| 9. Iceland 0.921 | 28. Czech Republic 0.878 | 47. Bahrain 0.824 |
| 10. Canada 0.920 | 29. Greece 0.866 | 48. Montenegro 0.807 |
| 11. United States 0.920 | 30. Brunei 0.865 | 49. Russia 0.804 |
| 12. Hong Kong 0.917 | 31. Estonia 0.865 | 50. Romania 0.802 |
| 13. New Zealand 0.915 | 32. Andorra 0.858 | 51. Kuwait 0.800 |
| 14. Sweden 0.913 | 33. Cyprus 0.856 | 52. Belarus 0.796 |
| 15. Liechtenstein 0.912 | 34. Malta 0.856 | 53. Oman 0.796 |
| 16. United Kingdom 0.909 | 35. Qatar 0.856 | 54. Barbados 0.795 |
| 17. Japan 0.903 | 36. Poland 0.855 | 55. Uruguay 0.795 |
| 18. South Korea 0.901 | 37. Lithuania 0.848 | 56. Bulgaria 0.794 |
| 19. Israel 0.899 | 38. Chile 0.847 | 57. Kazakhstan 0.794 |
| | | 58. Bahamas 0.792 |

| | | |
|-------------------------------------|--|--------------------------------------|
| 59. Malaysia 0.789 | 87. Peru 0.740 | 116. Vietnam 0.683 |
| 60. Palau 0.788 | 88. Thailand 0.740 | 117. Philippines 0.682 |
| 61. Panama 0.788 | 89. Ecuador 0.739 | 118. El Salvador 0.680 |
| 62. Antigua and Barbuda 0.786 | 90. China 0.738 | 119. Bolivia 0.674 |
| 63. Seychelles 0.782 | 91. Fiji 0.736 | 120. South Africa 0.666 |
| 64. Mauritius 0.781 | 92. Mongolia 0.735 | 121. Kyrgyzstan 0.664 |
| 65. Trinidad and Tobago 0.780 | 93. Saint Lucia 0.735 | 122. Iraq 0.649 |
| 66. Costa Rica 0.776 | 94. Jamaica 0.730 | 123. Cape Verde 0.648 |
| 67. Serbia 0.776 | 95. Colombia 0.727 | 124. Morocco 0.647 |
| 68. Cuba 0.775 | 96. Dominica 0.726 | 125. Nicaragua 0.645 |
| 69. Iran 0.774 | 97. Suriname 0.725 | 126. Guatemala 0.640 |
| 70. Georgia 0.769 | 98. Tunisia 0.725 | 127. Namibia 0.640 |
| 71. Turkey 0.767 | 99. Dominican Republic 0.722 | 128. Guyana 0.638 |
| 72. Venezuela 0.767 | 100. Saint Vincent and the Grenadines 0.722 | 129. Micronesia 0.638 |
| 73. Sri Lanka 0.766 | 101. Tonga 0.721 | 130. Tajikistan 0.627 |
| 74. Saint Kitts and Nevis 0.765 | 102. World average 0.717 | 131. Honduras 0.625 |
| 75. Albania 0.764 | 103. Libya 0.716 | 132. India 0.624 |
| 76. Lebanon 0.763 | 104. Belize 0.706 | 133. Bhutan 0.607 |
| 77. Mexico 0.762 | 105. Samoa 0.704 | 134. East Timor 0.605 |
| 78. Azerbaijan 0.759 | 106. Maldives 0.701 | 135. Vanuatu 0.597 |
| 79. Brazil 0.754 | 107. Uzbekistan 0.701 | 136. Congo, Republic of the 0.592 |
| 80. Grenada 0.754 | 108. Moldova 0.699 | 137. Equatorial Guinea 0.592 |
| 81. Bosnia and Herzegovina 0.750 | 109. Botswana 0.698 | 138. Kiribati 0.588 |
| 82. Macedonia 0.748 | 110. Gabon 0.697 | 139. Laos 0.586 |
| 83. Algeria 0.745 | 111. Paraguay 0.693 | 140. Bangladesh 0.579 |
| 84. Armenia 0.743 | 112. Egypt 0.691 | 141. Ghana 0.579 |
| 85. Ukraine 0.743 | 113. Turkmenistan 0.691 | 142. Zambia 0.579 |
| 86. Jordan 0.741 | 114. Indonesia 0.689 | 143. São Tomé and Príncipe 0.574 |
| | 115. Palestine 0.684 | 144. Cambodia 0.563 |
| | | 145. Nepal 0.558 |
| | | 146. Myanmar 0.556 |

- | | | |
|--------------------------------|------------------------|--|
| 147. Kenya 0.555 | 161. Comoros 0.497 | 176. Mali 0.442 |
| 148. Pakistan 0.550 | 162. Lesotho 0.497 | 177. Congo, Democratic Republic of the 0.435 |
| 149. Swaziland | 163. Senegal 0.494 | 178. Liberia 0.427 |
| 150. Syria 0.536 | 164. Haiti 0.493 | 179. Guinea Bissau 0.424 |
| 151. Angola 0.533 | 165. Uganda 0.493 | 180. Eritrea 0.420 |
| 152. Tanzania 0.531 | 166. Sudan 0.490 | 181. Sierra Leone 0.420 |
| 153. Nigeria 0.527 | 167. Togo 0.487 | 182. Mozambique 0.418 |
| 154. Cameroon 0.518 | 168. Benin 0.485 | 183. South Sudan 0.418 |
| 155. Papua New Guinea 0.516 | 169. Yemen 0.482 | 184. Guinea 0.414 |
| 156. Zimbabwe 0.516 | 170. Afghanistan 0.479 | 185. Burundi 0.404 |
| 157. Solomon Islands 0.515 | 171. Malawi 0.476 | 186. Burkina Faso 0.402 |
| 158. Mauritania 0.513 | 172. Ivory Coast 0.474 | 187. Chad 0.396 |
| 159. Madagascar 0.512 | 173. Djibouti 0.473 | 188. Niger 0.353 |
| 160. Rwanda 0.498 | 174. Gambia 0.452 | 189. Central African Republic 0.352 |
| | 175. Ethiopia 0.448 | |

Appendix D

List of countries by number of internet users

Extracted from [Wikipedia](#).

Values expressed as a percentage of the total population of the country.

- | | | |
|-----------------------------|----------------------------------|-----------------------------------|
| 1. Falkland Islands 99.02 % | 18. Sweden 91.51 % | 35. Anguilla 81.57 % |
| 2. Iceland 98.24 % | 19. United Arab Emirates 90.60 % | 36. Singapore 81.00 % |
| 3. Liechtenstein 98.09 % | 20. Netherlands 90.41 % | 37. Spain 80.56 % |
| 4. Bahrain 98.00 % | 21. Canada 89.84 % | 38. Slovakia 80.48 % |
| 5. Bermuda 98.00 % | 22. Germany 89.65 % | 39. Puerto Rico 80.32 % |
| 6. Andorra 97.93 % | 23. Switzerland 89.41 % | 40. The Bahamas 80.00 % |
| 7. Luxembourg 97.49 % | 24. New Zealand 88.47 % | 41. Latvia 79.89 % |
| 8. Norway 97.30 % | 25. Australia 88.24 % | 42. Israel 79.78 % |
| 9. Denmark 96.97 % | 26. Finland 87.70 % | 43. Taiwan 79.75 % |
| 10. Monaco 95.21 % | 27. Hong Kong 87.30 % | 44. Barbados 79.55 % |
| 11. Faroe Islands 95.11 % | 28. Estonia 87.24 % | 45. Hungary 79.26 % |
| 12. United Kingdom 94.78 % | 29. Niue 86.90 % | 46. Cayman Islands 79.00 % |
| 13. Gibraltar 94.44 % | 30. Belgium 86.52 % | 47. Malaysia 78.79 % |
| 14. Qatar 94.29 % | 31. France 85.62 % | 48. Kuwait 78.37 % |
| 15. Aruba 93.54 % | 32. Austria 84.32 % | 49. Azerbaijan 78.20 % |
| 16. South Korea 92.72 % | 33. Ireland 82.17 % | 50. Malta 77.29 % |
| 17. Japan 92.00 % | 34. Macau 81.64 % | 51. Guam 77.01 % |
| | | 52. Saint Kitts and Nevis 76.82 % |

| | | |
|---------------------------------------|--|--|
| 53. Kazakhstan 76.80 % | 81. Dominica 67.03 % | 109. Iran 53.23 % |
| 54. Czech Republic 76.48 % | 82. Uruguay 66.40 % | 110. Mauritius 53.23 % |
| 55. Russia 76.41 % | 83. Albania 66.36 % | 111. China 53.20 % |
| 56. United States 76.18 % | 84. Costa Rica 66.03 % | 112. Ukraine 52.48 % |
| 57. Lebanon 76.11 % | 85. Chile 66.01 % | 113. Paraguay 51.35 % |
| 58. Cyprus 75.90 % | 86. Jordan 62.30 % | 114. Tunisia 50.88 % |
| 59. Slovenia 75.50 % | 87. Armenia 62.00 % | 115. Georgia 50.00 % |
| 60. Brunei 75.00 % | 88. Dominican Republic 61.33 % | 116. Cabo Verde 48.17 % |
| 61. Lithuania 74.38 % | 89. Italy 61.32 % | 117. Gabon 48.05 % |
| 62. New Caledonia 74.00 % | 90. Palestinian Authority 61.18 % | 118. Thailand 47.50 % |
| 63. Saudi Arabia 73.75 % | 91. Venezuela 60.00 % | 119. Uzbekistan 46.79 % |
| 64. Poland 73.30 % | 92. Bulgaria 59.83 % | 120. Saint Lucia 46.73 % |
| 65. Trinidad and Tobago 73.30 % | 93. U.S. Virgin Islands 59.61 % | 121. Fiji 46.51 % |
| 66. Antigua and Barbuda 73.00 % | 94. Mexico 59.54 % | 122. Vietnam 46.50 % |
| 67. Croatia 72.70 % | 95. Romania 59.50 % | 123. Tuvalu 46.01 % |
| 68. Macedonia 72.16 % | 96. Maldives 59.09 % | 124. Peru 45.46 % |
| 69. Belarus 71.11 % | 97. Turkey 58.35 % | 125. Suriname 45.40 % |
| 70. Moldova 71.00 % | 98. Morocco 58.27 % | 126. Jamaica 45.00 % |
| 71. Brazil 70.5 % | 99. Colombia 58.14 % | 127. Belize 44.58 % |
| 72. Portugal 70.42 % | 100. Seychelles 56.51 % | 128. Algeria 42.95 % |
| 73. Argentina 70.15 % | 101. Grenada 55.86 % | 129. Bhutan 41.77 % |
| 74. Montenegro 69.88 % | 102. Saint Vincent and the Grenadines 55.57 % | 130. Jersey 41.03 % |
| 75. Oman 69.82 % | 103. Philippines 55.50 % | 131. Ascension 41.0 % |
| 76. Bosnia and Herzegovina 69.33 % | 104. Montserrat 54.60 % | 132. Tonga 39.95 % |
| 77. Greece 69.09 % | 105. San Marino 54.21 % | 133. Bolivia 39.70 % |
| 78. Greenland 68.50 % | 106. Ecuador 54.06 % | 134. Egypt 39.21 % |
| 79. French Polynesia 68.44 % | 107. South Africa 54.00 % | 135. Cuba 38.77 % |
| 80. Serbia 67.06 % | 108. Panama 54.00 % | 136. British Virgin Islands 37.60 % |
| | | 137. Saint Helena 37.6 % |
| | | 138. Guyana 35.66 % |
| | | 139. Ghana 34.67 % |

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|--|--------------------------------|--|
| 140. Guatemala 34.51 % | 166. Yemen 24.58 % | 192. Benin 11.99 % |
| 141. Kyrgyzstan 34.50% | 167. Nicaragua 24.57 % | 193. Sierra Leone 11.77 % |
| 142. Micronesia, Federated States of 33.35 % | 168. Vanuatu 24.00 % | 194. Togo 11.31 % |
| 143. Sri Lanka 32.05 % | 169. Equatorial Guinea 23.78 % | 195. Mali 11.11 % |
| 144. Syria 31.87 % | 170. Zimbabwe 23.12 % | 196. Solomon Islands 11.00 % |
| 145. Namibia 31.03 % | 171. Mongolia 22.27 % | 197. Afghanistan 10.6 % |
| 146. Honduras 30.00 % | 172. Uganda 21.88 % | 198. Guinea 9.80 % |
| 147. Marshall Islands 29.79 % | 173. Laos 21.87 % | 199. Malawi 9.61 % |
| 148. India 29.55 % | 174. Iraq 21.23 % | 200. Papua New Guinea 9.60 % |
| 149. Samoa 29.41 % | 175. Tajikistan 20.47 % | 201. Wallis and Futuna 8.95 % |
| 150. El Salvador 29.00 % | 176. Libya 20.27 % | 202. Republic of the Congo 8.12 % |
| 151. Swaziland 28.57 % | 177. Rwanda 20.00 % | 203. Comoros 7.94 % |
| 152. Sudan 28.00 % | 178. Nepal 19.69 % | 204. Liberia 7.32 % |
| 153. São Tomé and Príncipe 28.00 % | 179. The Gambia 18.50 % | 205. South Sudan 6.70 % |
| 154. Botswana 27.50 % | 180. Bangladesh 18.25 % | 206. Democratic Republic of the Congo 6.21 % |
| 155. Lesotho 27.36 % | 181. Mauritania 18.00 % | 207. Burundi 5.17 % |
| 156. Ivory Coast 26.53 % | 182. Turkmenistan 17.99 % | 208. Chad 5.00 % |
| 157. Kenya 26.00 % | 183. Mozambique 17.52 % | 209. Madagascar 4.71 % |
| 158. Nigeria 25.67 % | 184. Pakistan 15.51 % | 210. Niger 4.32 % |
| 159. Senegal 25.66 % | 185. Ethiopia 15.37 % | 211. Central African Republic 4.00 % |
| 160. Cambodia 25.57 % | 186. Burkina Faso 13.96 % | 212. Guinea-Bissau 3.76 % |
| 161. Zambia 25.51 % | 187. Kiribati 13.70 % | 213. Somalia 1.88 % |
| 162. Indonesia 25.37 % | 188. Djibouti 13.13 % | 214. Eritrea 1.18 % |
| 163. Timor Leste 25.25 % | 189. Tanzania 13.00 % | |
| 164. Myanmar 25.07 % | 190. Angola 13.00 % | |
| 165. Cameroon 25.00 % | 191. Haiti 12.23 % | |